Optimal Communication Complexity of Authenticated Byzantine Agreement

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Byzantine Agreement (BA)

A set of parties $\{r_1, .., r_n\}$ have input values $\{x_1, .., x_n\}$, and agree on a single output .

- At most f parties are faulty and behave arbitrarily — Byzantine fault.
- Consistency. Honest parties do not output different values.
- Termination. Every honest party eventually outputs a value.
- Validity. If every honest party has the same input value ,
  every honest party outputs the value $y = b$ — Unanimity.

Byzantine Agreement (BA)

Unauthenticated model.

- No cryptography, i.e., information-theoretic security.
- $f < n/3$ is the best possible.

Authenticated model.

- Assume cryptography, e.g., digital signature with PKI.
- $f < n/2$ is the best possible.

Communication Complexity

The maximum amount of bits transferred by all honest parties combined across all executions–worst-case communication cost.

- All parties multicast $O(1)$ messages, i.e., all-to-all communication

  $\rightarrow O(n^2)$ communication

- All-to-all communication with $O(n)$ messages (e.g., a quorum of votes)

  $\rightarrow O(n^3)$ communication

Communication Complexity of BA

Model	Fault-tolerace	Lower Bound	Upper Bound
unauthenticated	$f < n/3$	$\Omega(n^2)$ [Dolev-Resichuk]	$O(n^2)$ [Berman et al.]
authenticated (PKI)	$f < n/2$		$O(n^3)$ [Dolev-Strong]

Communication Complexity of BA

> 0 : any constant

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authenticated (PKI)	$f < n/2$		$O(n^3)$ [Dolev-Strong]
authenticated (trusted setup)	$f < n/2$		$O(n^2)$ this work
authenticated (PKI)	$f < (1/2 - \varepsilon)n$		$O(n^2)$ this work

Other Assumptions

Lockstep synchrony model.

- Every party runs at the same clock speed
  → a clock step is called round

- All message sent by honest parties are delivered by the next round

Adaptive corruption.

- An adversary can corrupt parties anytime in the protocol execution

Outline

1. Achieving BA from Graded Agreement (GA)

- Berman et al's protocol is a problem reduction from BA to GA.

2. Solving GA for $f \geq n/3$

- Solution 1: GA with $f < n/2$ and trusted setup.
- Solution 2: GA with $f \leq (1/2 - \varepsilon)n$ and PKI.

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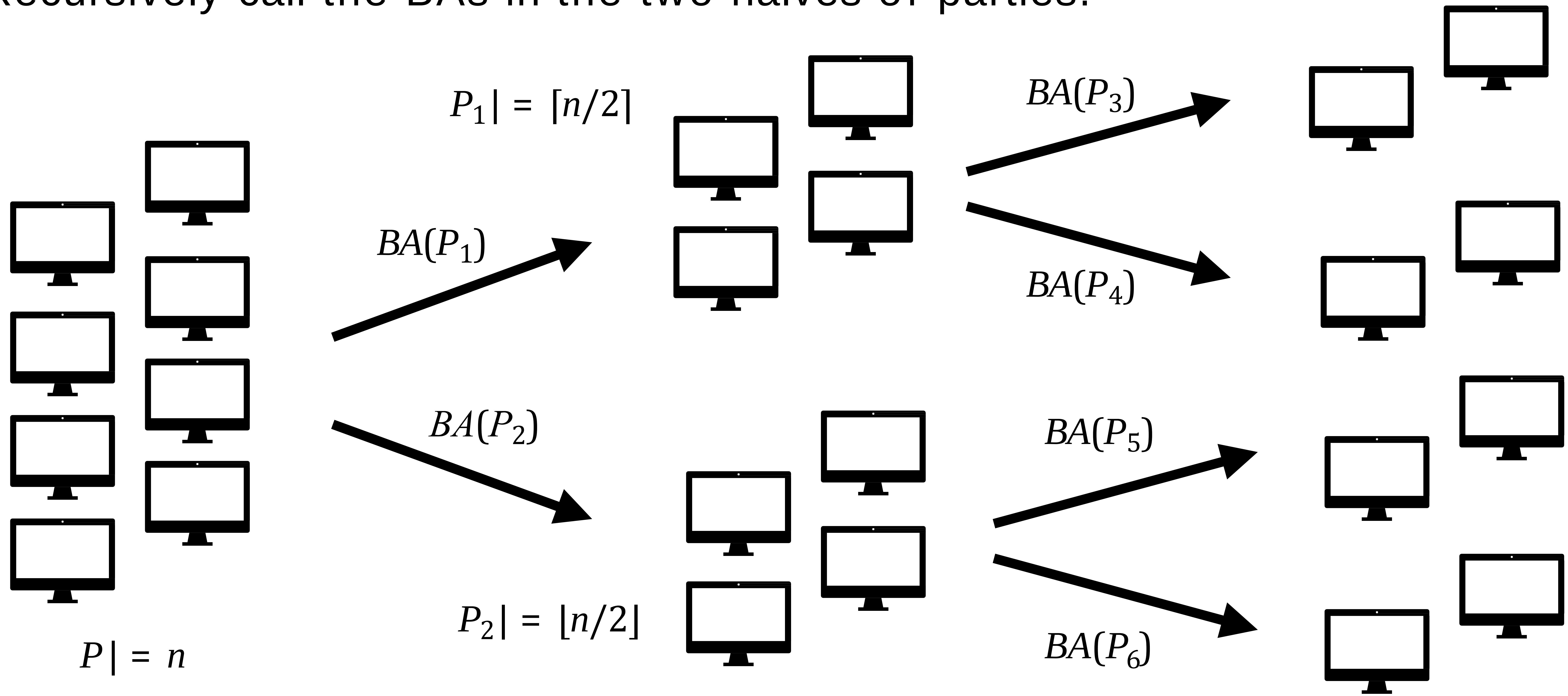
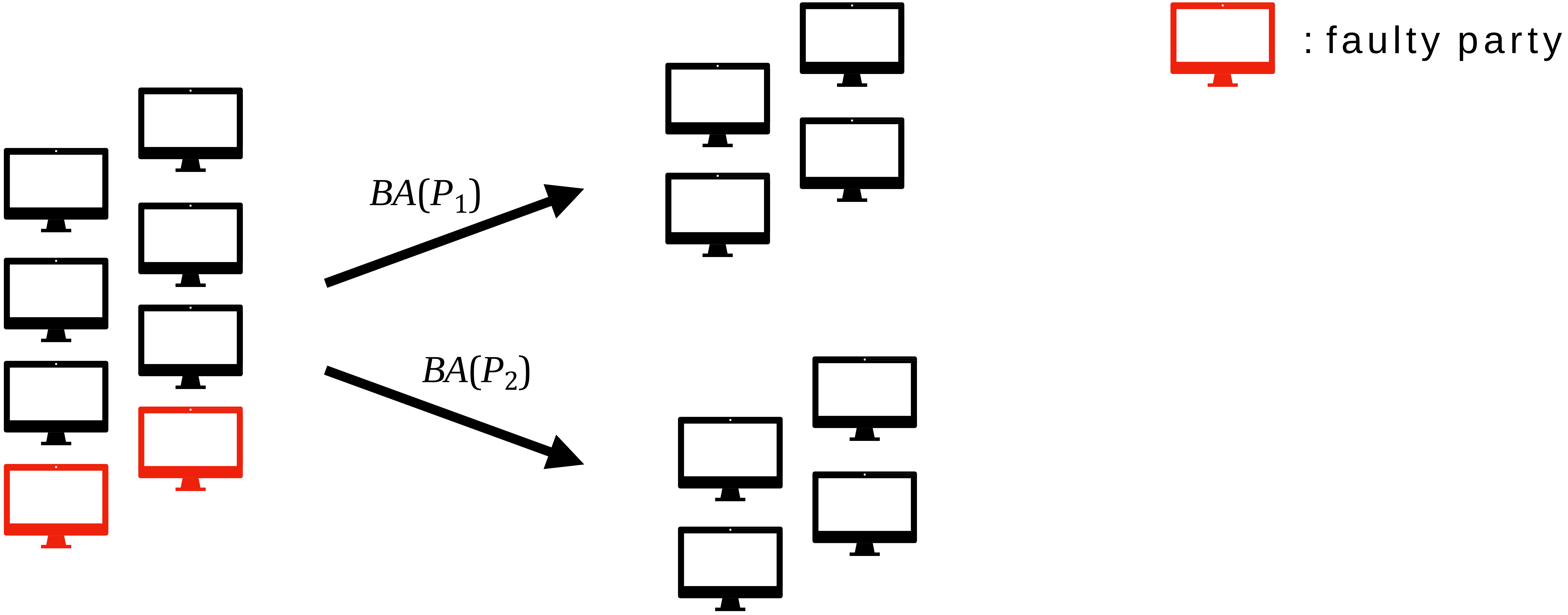
Berman et al.

Recursively call the BAs in the two halves of parties.



Berman et al.

One of two halves preserves the 1/3 fault fraction → “correct” BA exec.



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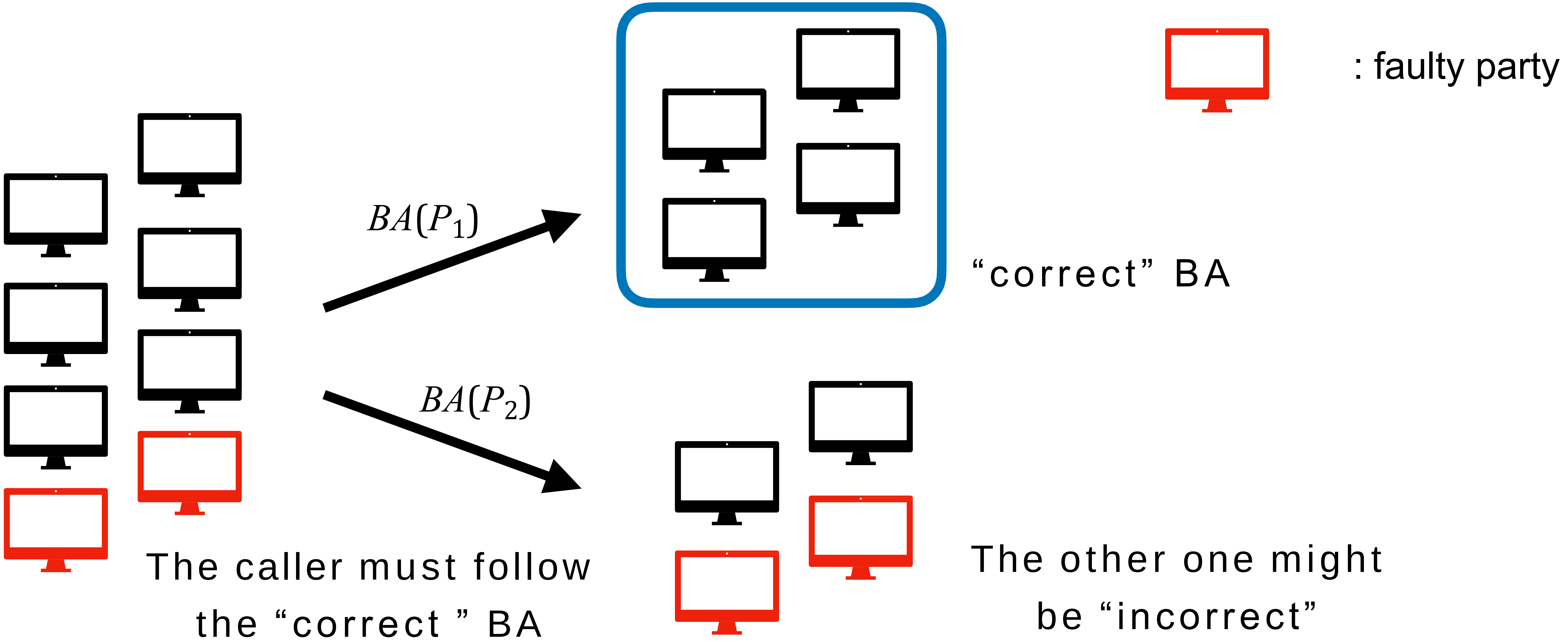
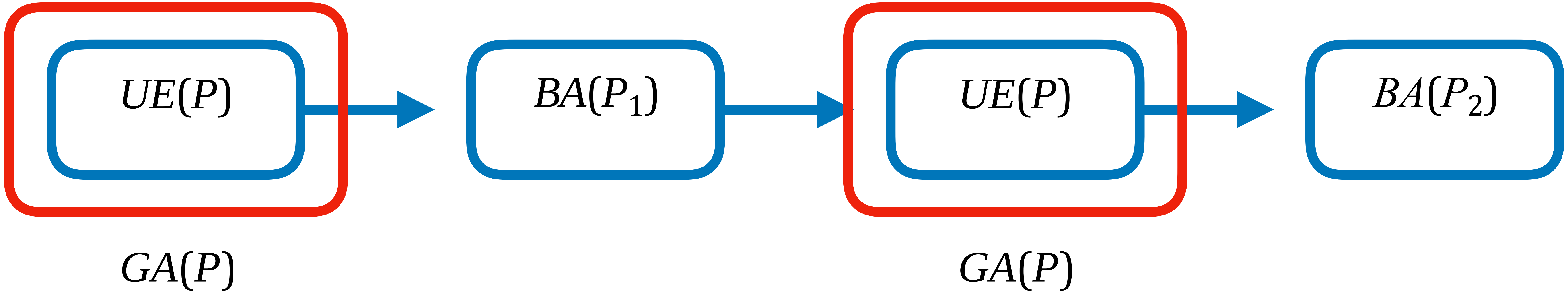


Berman et al.

The tailored Universal Exchange pre-process (for $f < n/3$) helps parties ignore the incorrect BA and follow the correct BA.



What the Universal Exchange achieves is the well-known problem called Graded Agreement.

Graded Agreement (GA)

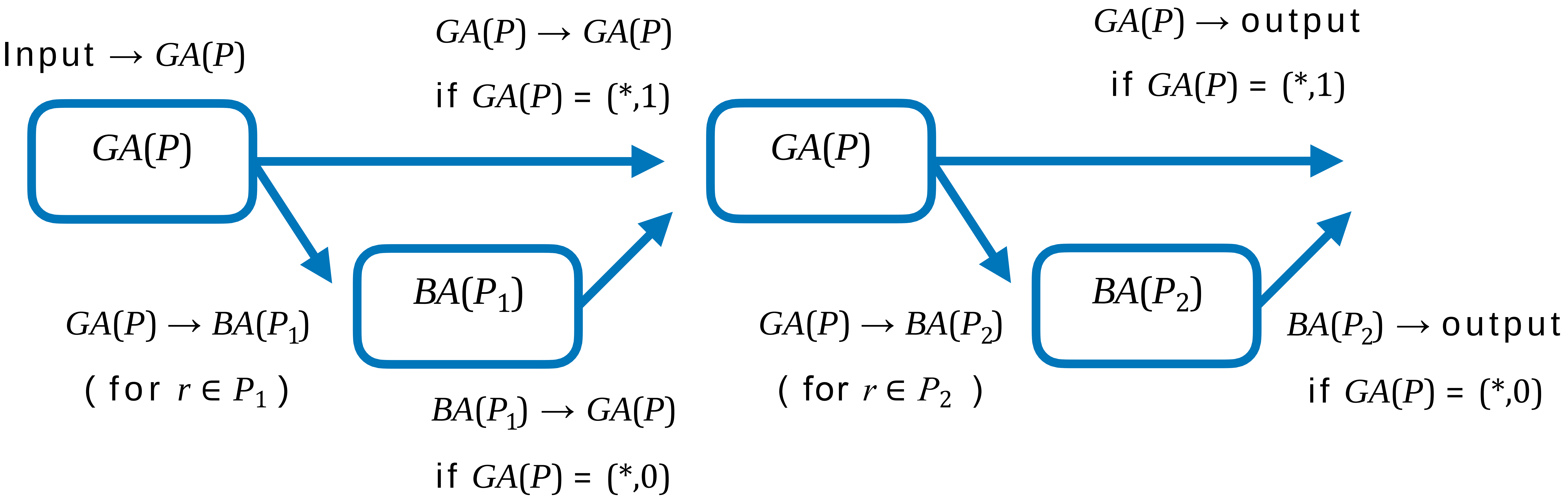
A set of parties $\{r_1, .., r_n\}$ have input values $\{x_1, .., x_n\}$, and each party outputs a pair (y, g) of value and a grade bit $g \in \{0,1\}$

- Consistency. If an honest party outputs $(y, 1)$, every honest party outputs $(y, *)$
- Validity. If every honest party has the same input value $x_i, .. x_j = b$, every honest party outputs $(b, 1)$
- Termination. Every honest party eventually outputs a pair.

BA in partition (i.e., $BA(P)$)

1. GA determines the input of BA

2. If GA outputs grade 1, ignore BA's output.

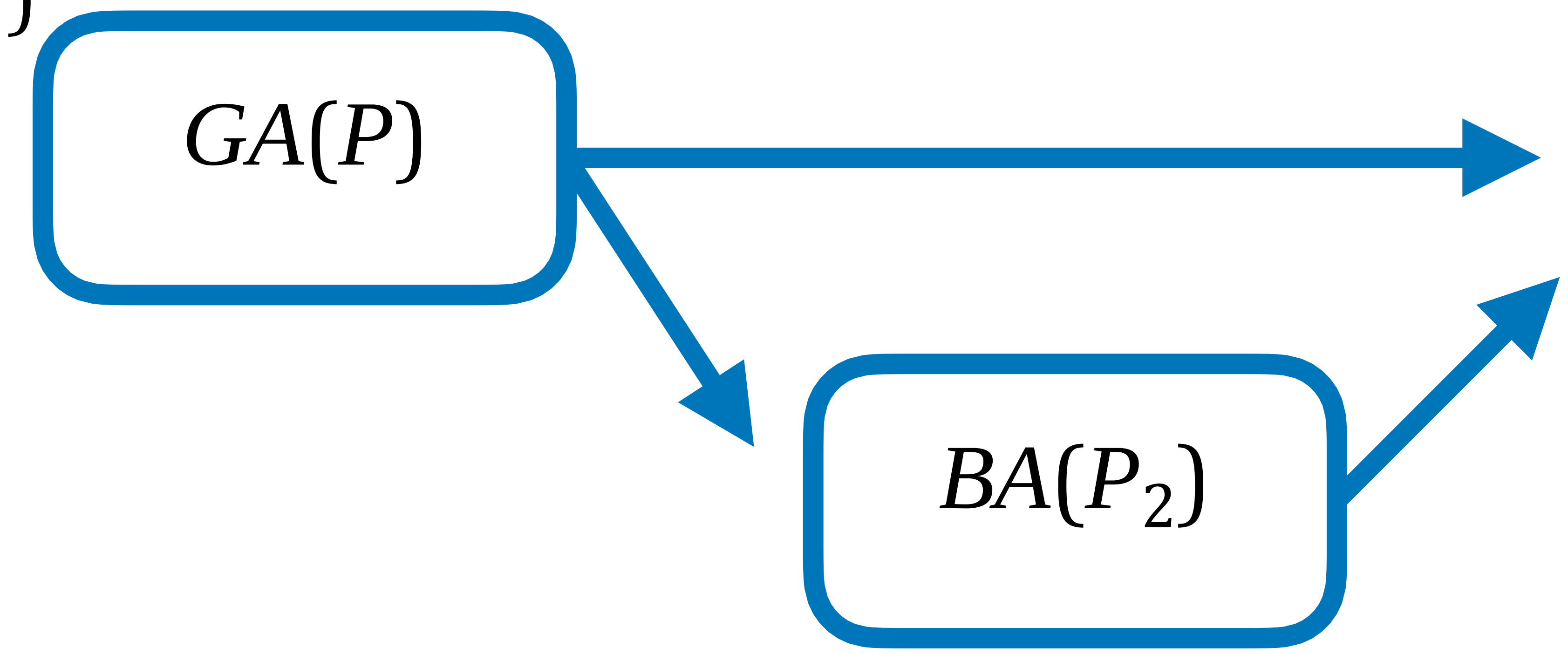


Idea 1: An agreed upon value will not be changed.

If all honest parties already agree on a value at the beginning of each step, they do not change the value in the step.

Everyone inputs

$\to GA(P)$



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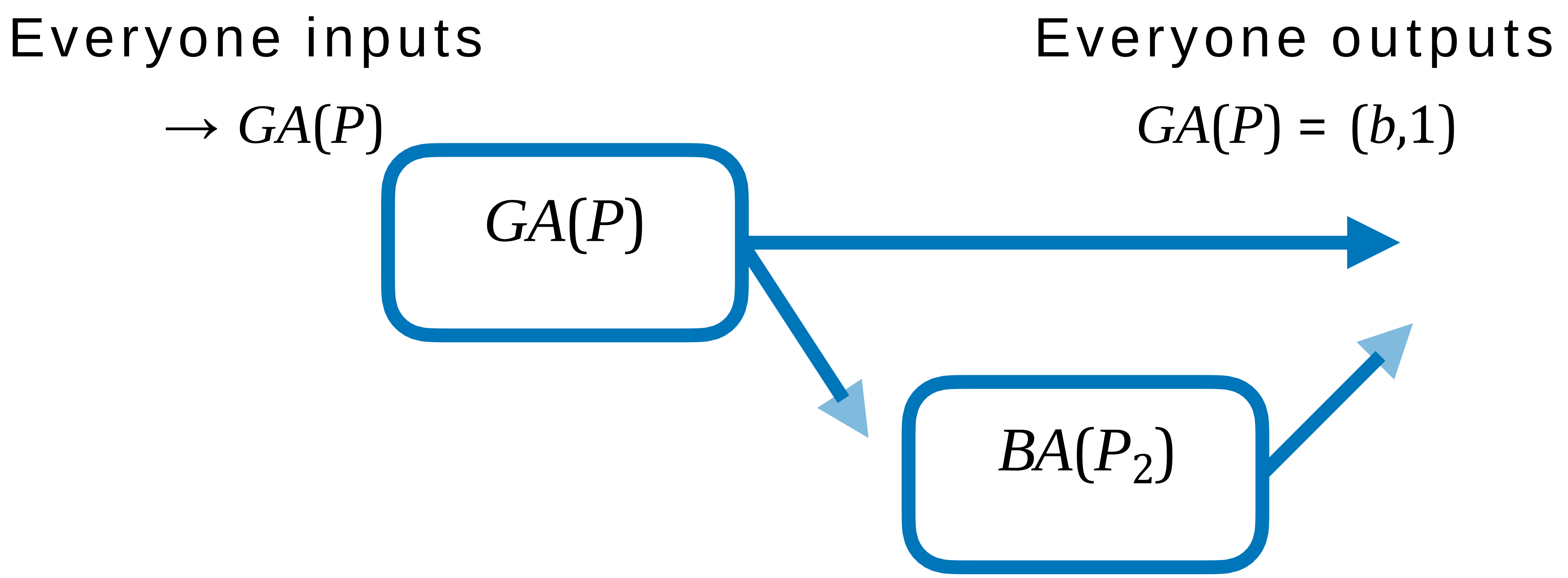
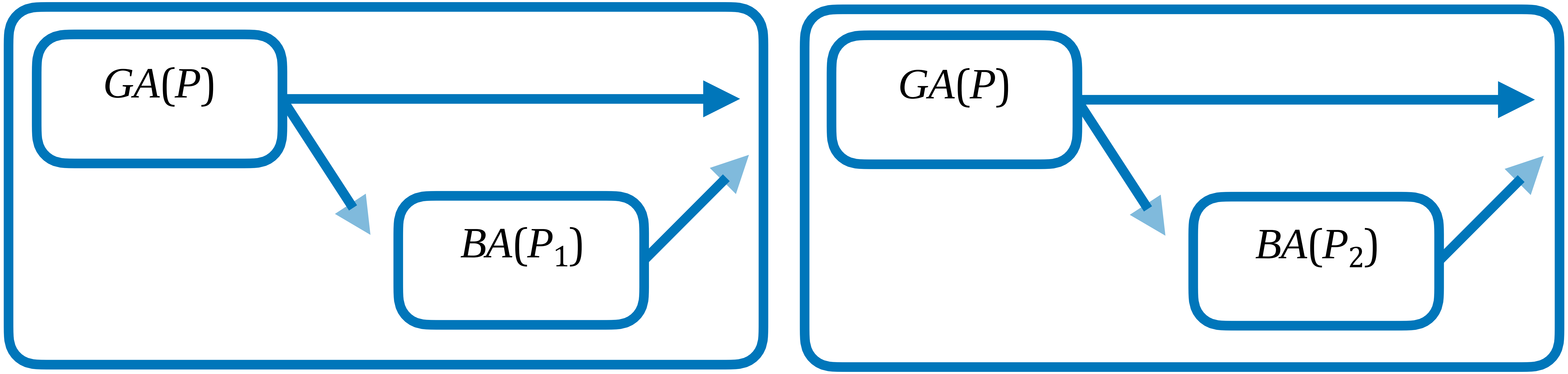


Validity

If all honest parties input the same value, they all output the value.

Input $\rightarrow GA(P)$ $\rightarrow$ Output



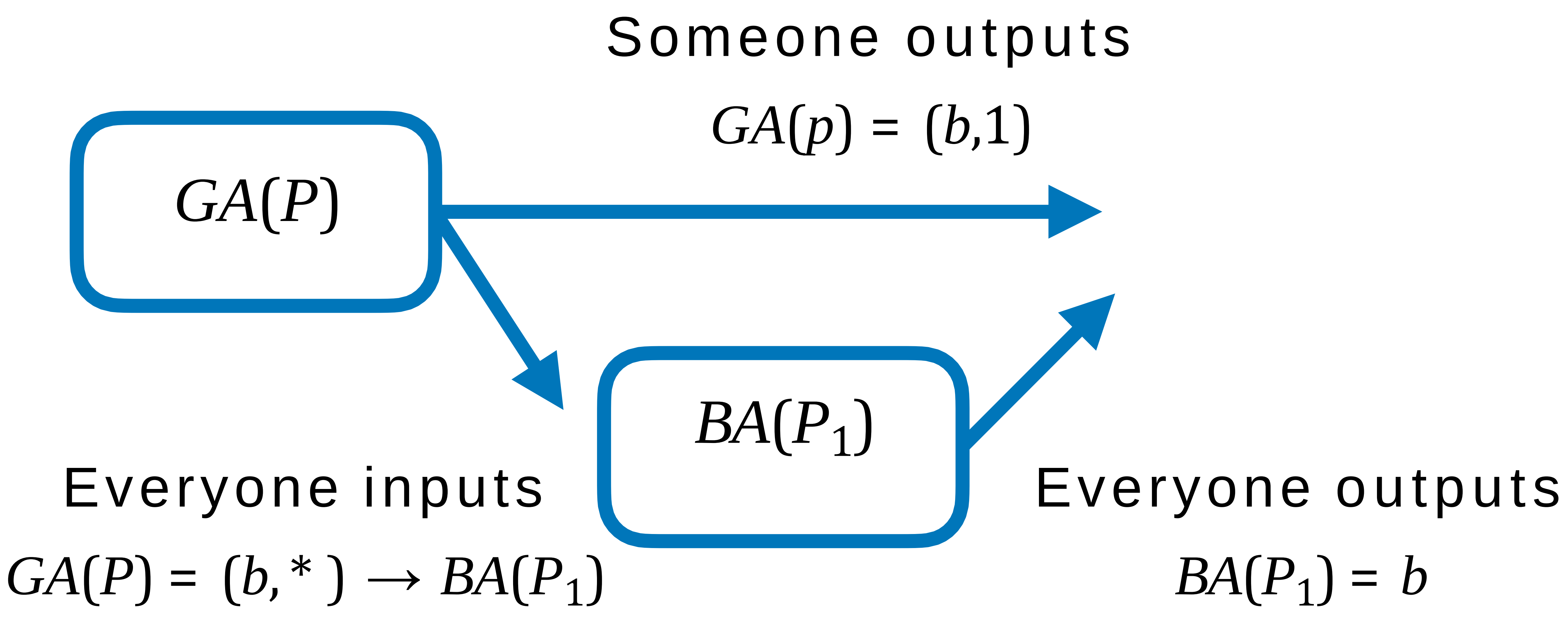
Step 1: Everyone outputs the agreed upon value

Step 2: Everyone outputs the agreed upon value

Idea 2: The "correct" step drives agreement

All honest parties agree on a value at the end of the "correct" step.

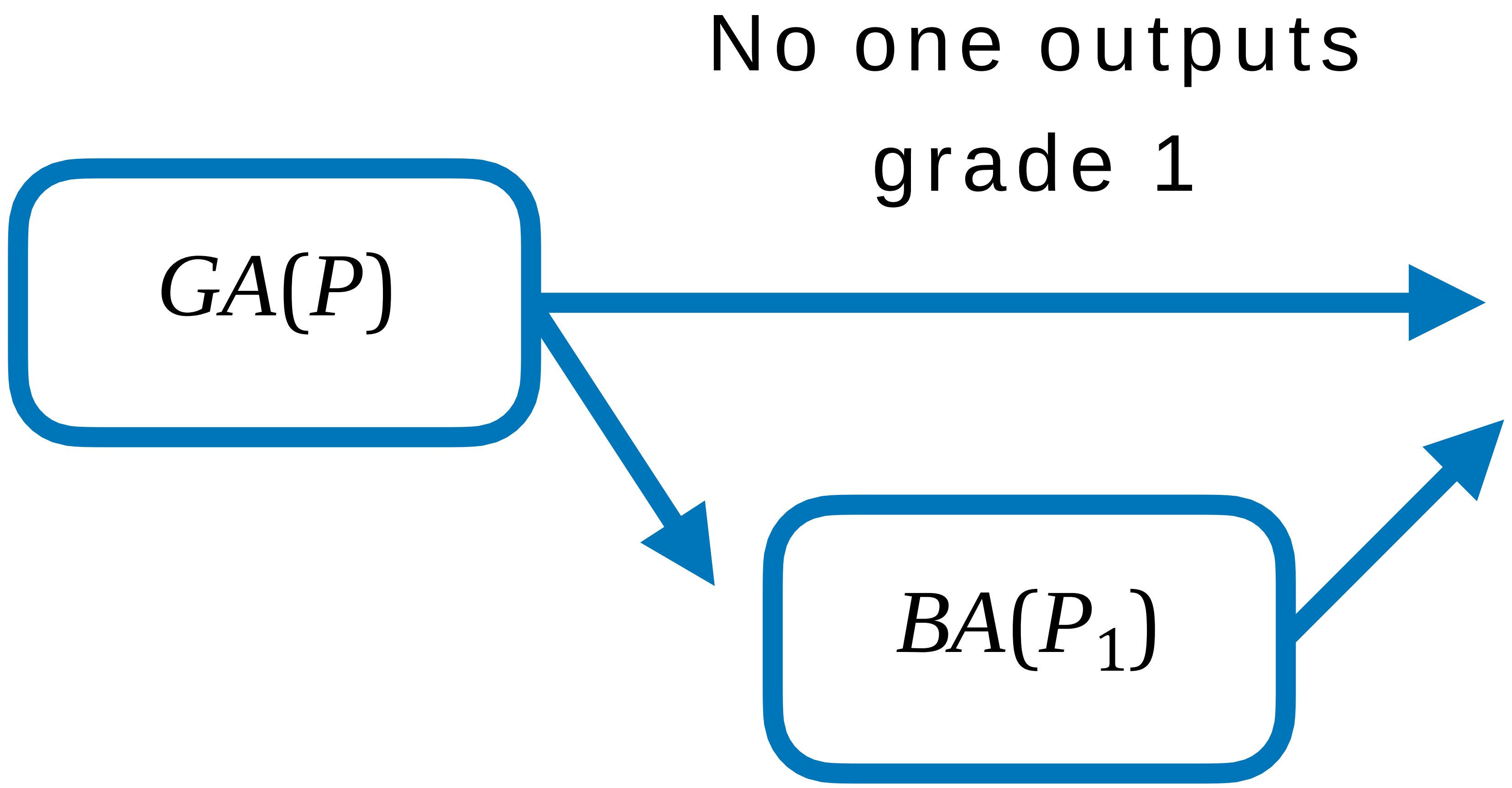
Case 1: Someone outputs a value with grade 1 in GA.



Idea 2: The “correct” step drives agreement

All honest parties agree on a value at the end of the “correct” step.

Case 2: No one outputs grade 1 in GA.



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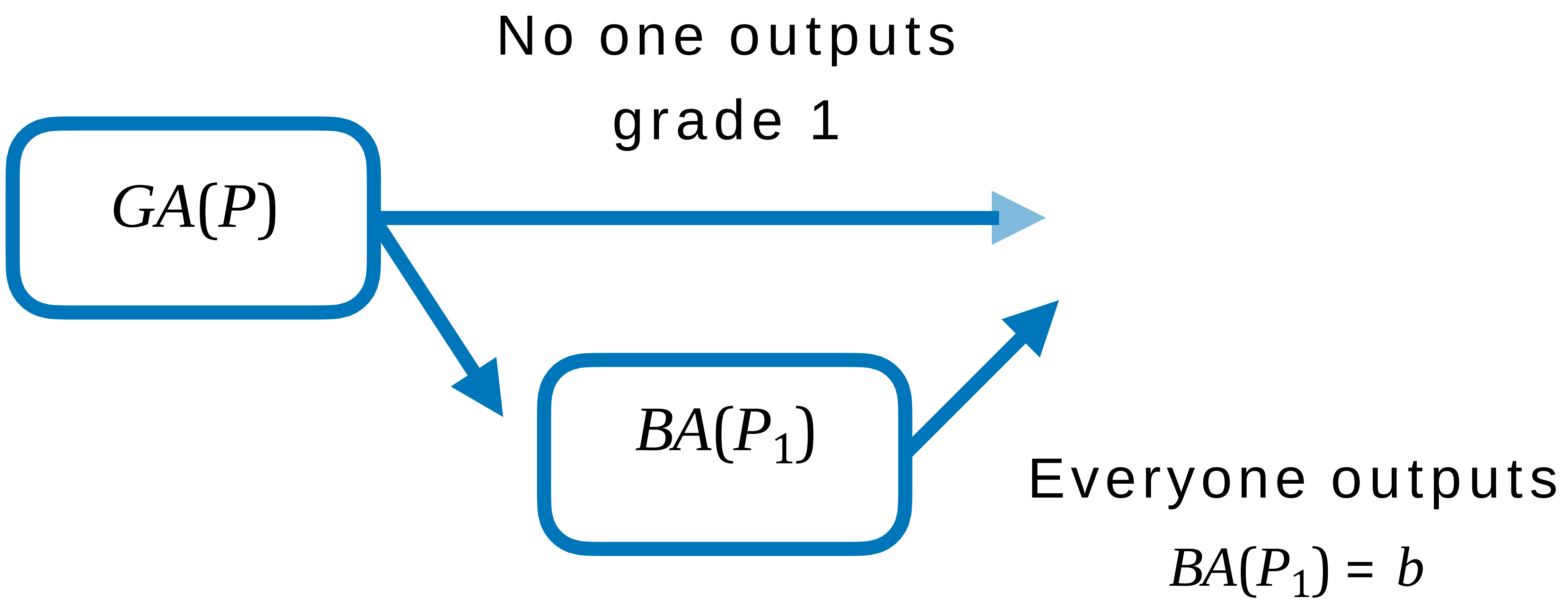
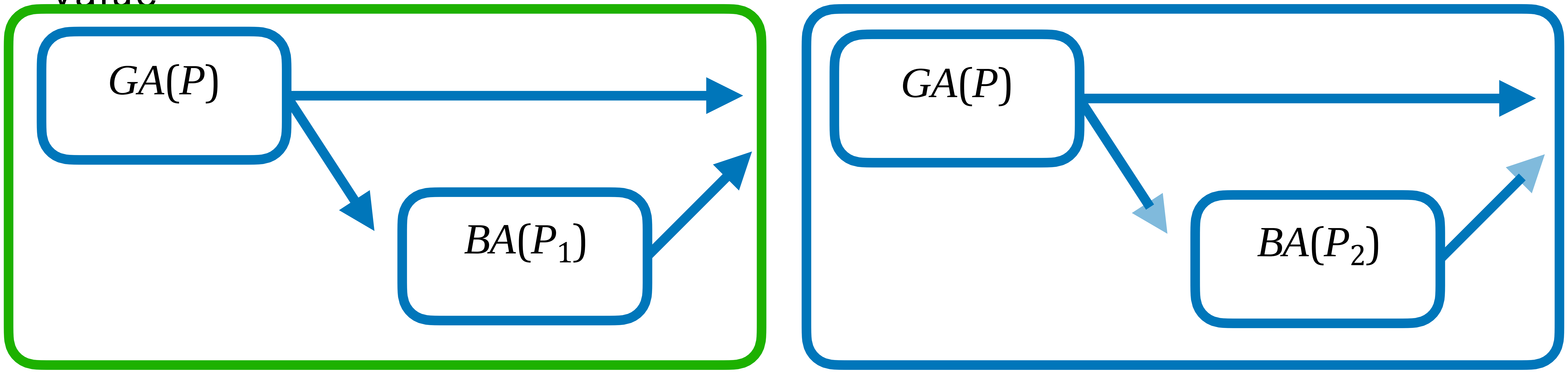


Consistency (case 1: step 1 is correct)

The "correct" first step drives agreement, and the second step does not change the agreed upon value.

Step 1: Everyone agrees on a value

Step 2: Everyone outputs the agreed upon value

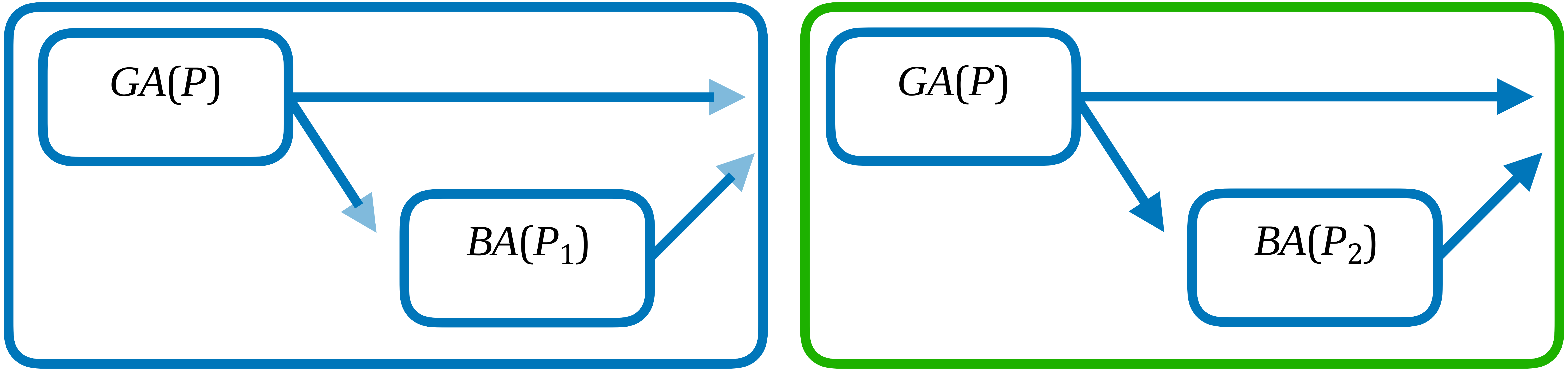


Step 1 is "correct"

Consistency (case 2: step 2 is correct)

All honest parties agree on a value following the “correct” second step.

Step 2: Everyone agrees on a value



Step 2 is “correct”

Communication Complexity

If the GA protocol costs $O(n^2)$ communication, the total communication of the BA protocol will be $O(n^2)$

k - the depth of the recursion

$$C(n) \leq O\left(2^k + \sum_{i=0}^{k} 2^i \cdot \left(\frac{n}{2^i}\right)^2\right) = O(n^2)$$

The end of the recursion has 2^k partitions of $O(1)$ parties

Depth recursion has 2^i GAs with $O((\frac{n}{2^i})^2)$ communication

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2. Solving GA for $f \geq n/3$

- Solution 1: GA with $f < n/2$ and trusted setup.
- Solution 2: GA with $f \leq (1/2 - \varepsilon)n$ and PKI.

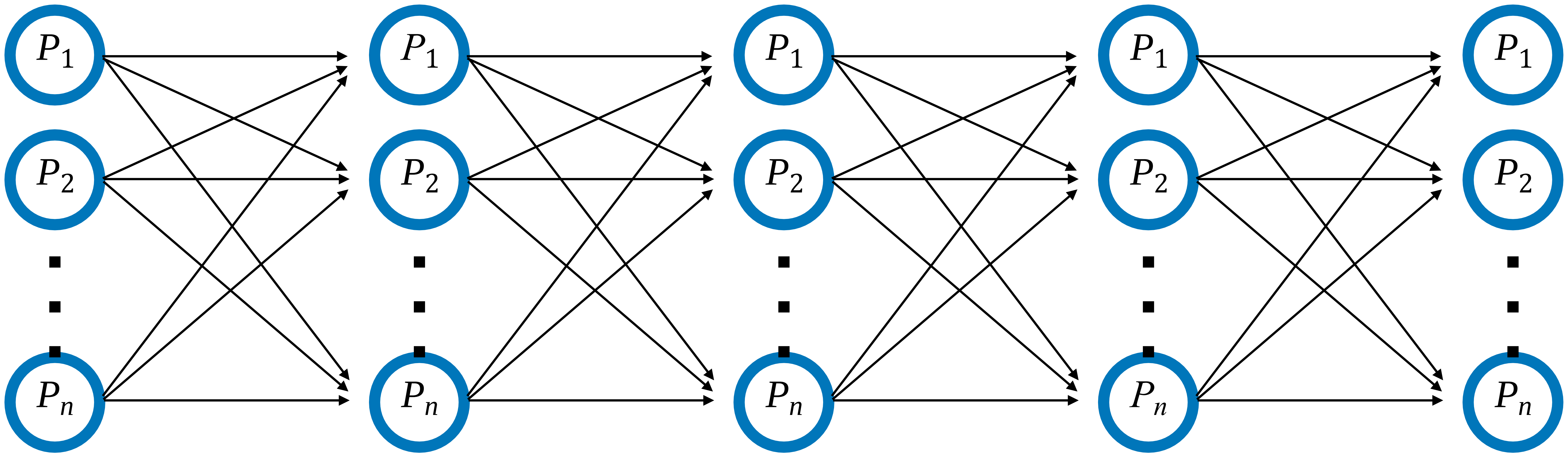
Warmup: GA with $f < n/2$

$C^1(b)$: $n - f$ (vote1, b)

$C^2(b)$: $n - f$ (vote2, b)

2. Forward $C^1(b)$

4. Forward $C^2(b)$



1. Send the input to all as (vote1, b)

3. If no $C^1(b')$ for $b' \neq b$, send (vote2, b) to all

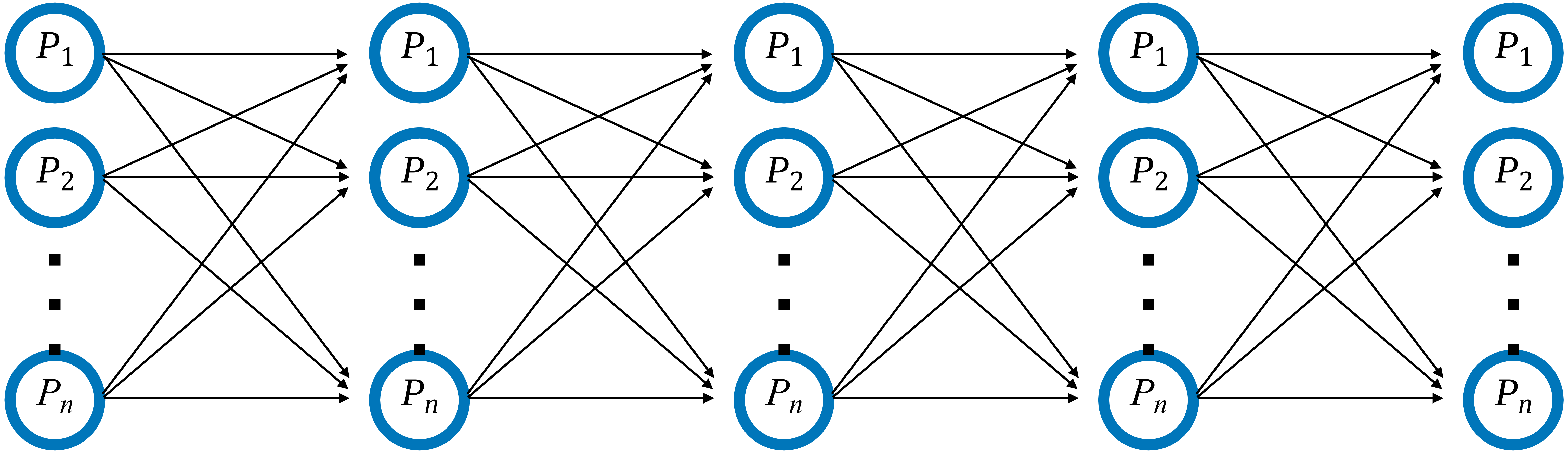
If it receives $C^2(b)$ output

(1) in round 3 $\Rightarrow g \leftarrow 1$

(2) in round 4 $\Rightarrow g \leftarrow 0$

Core idea: Eliminate conflicting majority vote2

Two different majority vote2 $C^2(b)$ and $C^2(b')$ cannot be collected.



2. Forward $C^1(b)$

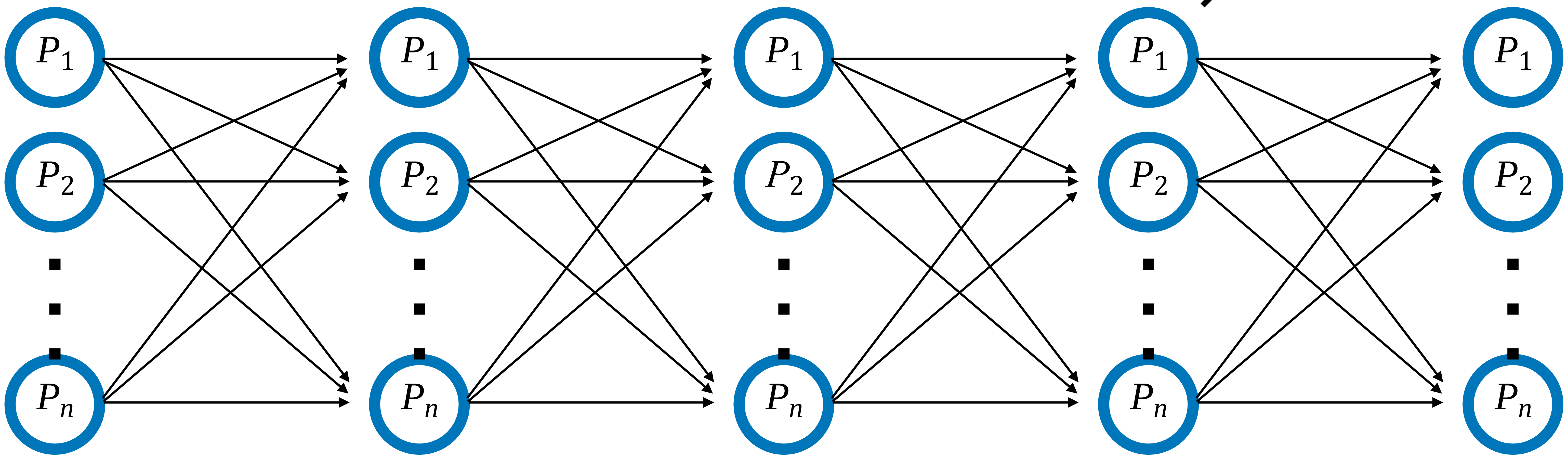
3. If no $C^1(b')$ for $b' \neq b$,

send $(\text{vote2}, b)$ to all

Consistency

If someone outputs $(b,1)$, everyone outputs $(b,*)$.

Forward
$C^2(b)$

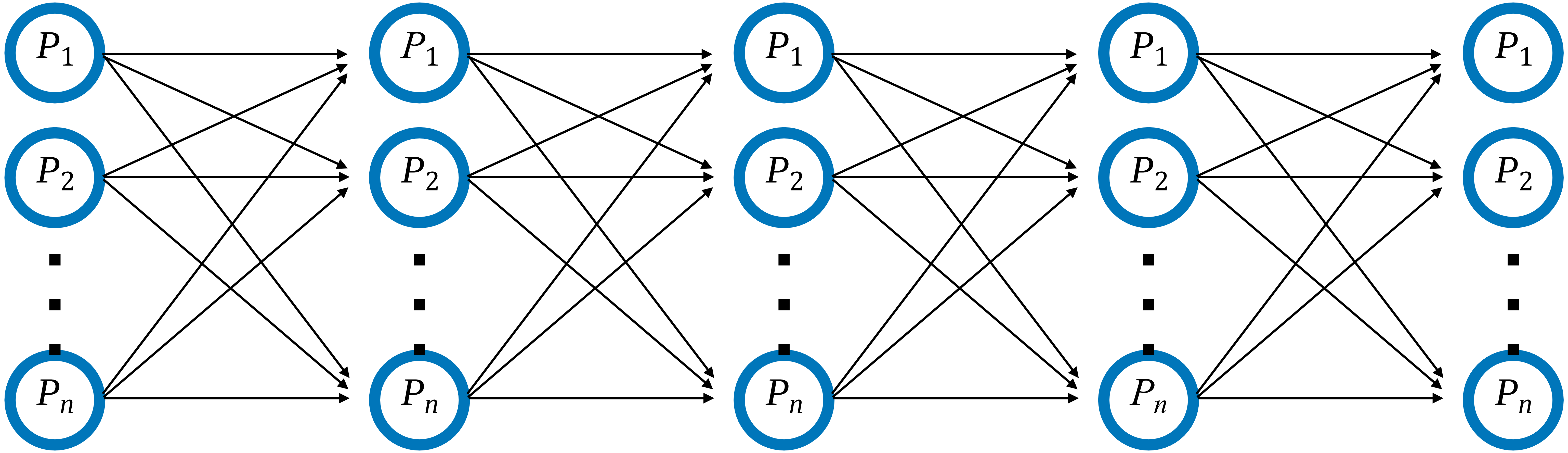


Everyone receives $C^2(b)$

$\Rightarrow$ everyone outputs $(b,*)$

The communication complexity is $\Omega(n^3)$

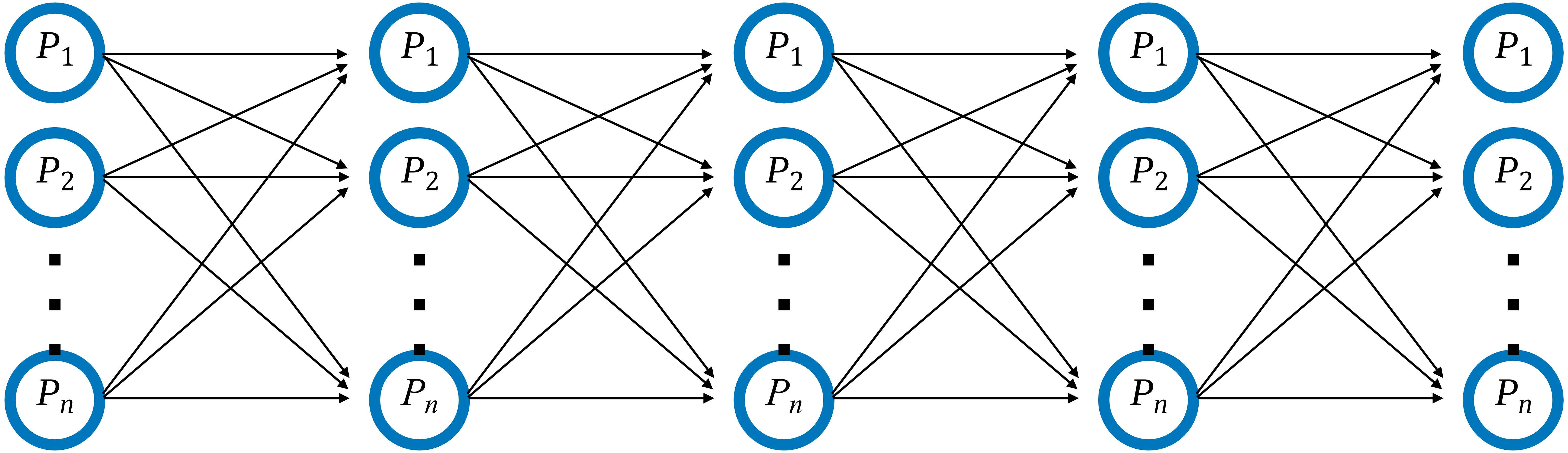
Everyone forwarding $n - f = \Omega(n)$ vote2 costs $\Omega(n^3)$ communication



2. Forward $C^1(b)$

Solution 1: Combining a set of votes

Combining $C^1(b)$ into a single signature using $(n - f, n)$-threshold signature



2. Combine $C^1(b)$ into a single signature, and forward it

Threshold signatures require strong trusted key setup assumption

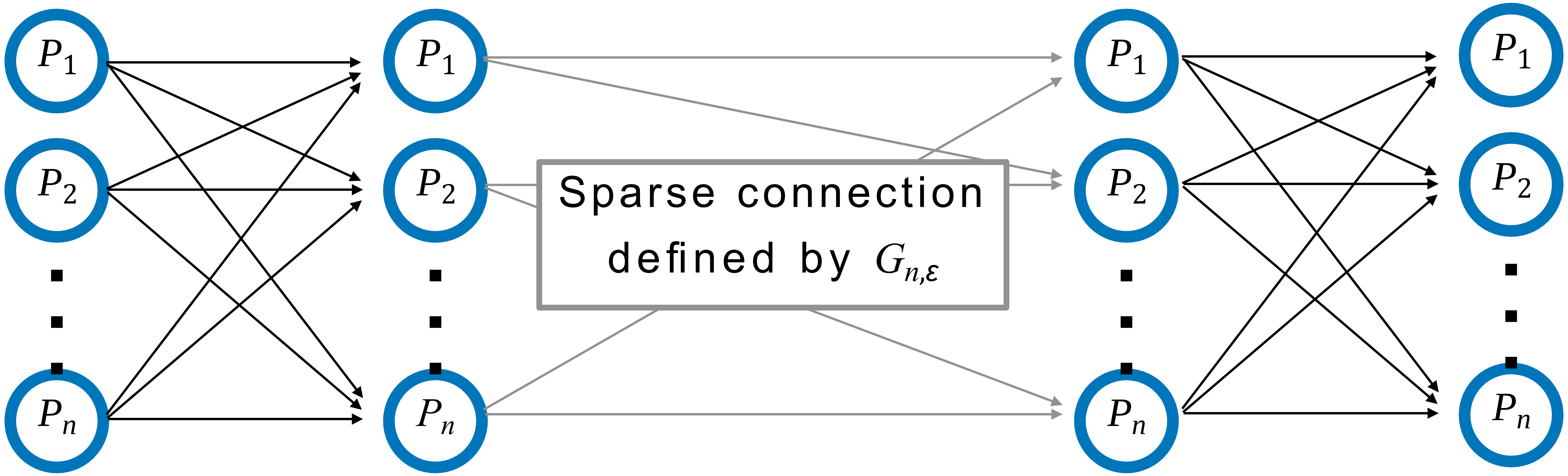
Solution 2: Expander

(n, α, β)-expander. $(0 < \alpha, \beta < 1)$

- A graph of nodes with good connectivety.
- Expansion property. For any subset of nodes, the neighbors $\Gamma(S)$ contains more than βn nodes.
- For any $0 < \alpha, \beta < 1$, a constant degree (n, α, β)-expander exists.
- We use $(n, 2\varepsilon, 1 - 2\varepsilon)$-expander denoted $G_{n,\varepsilon}$

Solution 2: GA with $f \le (1/2 - \varepsilon)n$

1. Send the input to all as (vote1, b)
2. Send $C^1(b)$ to the neighbors in $G_{n,\varepsilon}$
3. If no $C^1(b')$ for $b' \neq b$, send $(\text{vote2}, b)$ to all



The degree of $G_{n,\varepsilon}$ is $O(1)$

→ parties can forward $\Omega(n)$-sized $C^1(b)$ with $O(n^2)$ total communication

Solution 2: GA with $f \leq (1/2 - \varepsilon)n$

Suppose $C^2(b)$ is collected.

→ At least $n - f \geq f + 2\epsilon n$ parties, i.e., 2ε honest parties, must have sent vote2 on the value , who must have propagated $C^1(b)$ to the neighbors in $G_{n,\varepsilon}$

→ More than $(1 - 2\varepsilon)n \geq 2f$ parties, i.e., $> f$ honest parties, must have received $C^1(b)$, who could not have sent vote2 on $b' \neq b$

→ $C^2(b')$ cannot be collected.

Summary

- Solution 1 achieves $f < n/2$, but requires trusted key setup for threshold signatures.
- Solution 2 tolerate $f \le (1/2 - \varepsilon)n$, but requires only PKI.

<table>
<tr><td>authenticated
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